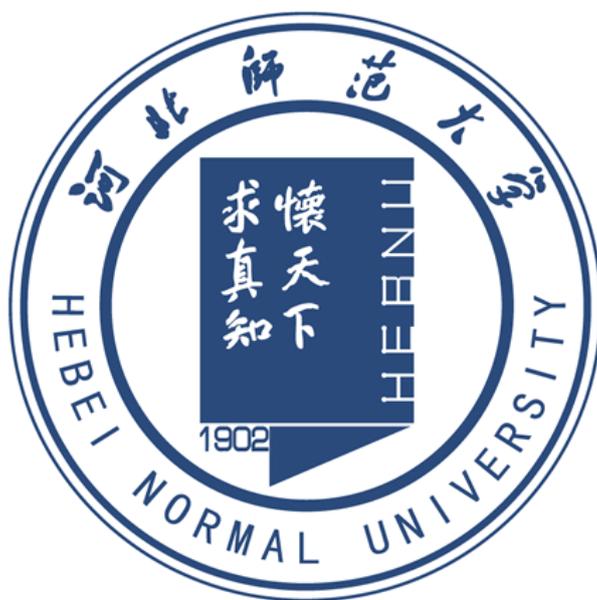


# Recent Trends in the Theory of Power Semigroups

*half-day workshop hosted by*

School of Mathematical Sciences, Hebei Normal University

21 September, 2025



*organized by*

Salvatore TRINGALI, Liping YUAN (Dean), Kerou WEN (2nd-year graduate), Zhengliang YUAN (1st-year graduate student), Lingxi LI (4th-year undergraduate), and Chen WANG (2nd-year undergraduate)

*supported by*

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## Schedule

- 14:00 – 14:25** Salvatore TRINGALI (Hebei Normal University, China)  
*Recent trends in the theory of power semigroups*  
(opening talk by the organizers)
- 14:30 – 15:15** Felix GOTTI (MIT, United States of America)  
*Maximal Common Divisors in Power Monoids*
- 15:20 – 16:05** Laura COSSU (University of Cagliari, Italy)  
*Power Monoids in a New Framework for Factorization*
- 16:05 – 16:20** **Coffee/Tea Break**
- 16:20 – 17:05** Pedro A. GARCÍA-SÁNCHEZ (University of Granada, Spain)  
*Isomorphism problems for ideals of numerical semigroups* (online)
- 17:10 – 17:55** Balint RAGO (University of Graz, Austria)  
*On power monoids and their automorphisms* (online)
- 18:00 – 18:25** Andreas REINHART (University of Graz, Austria)  
*Power monoids and their system of length sets* (online)

## Practical info

All times are in Chinese Standard Time (CST). Check your local time at the link

<https://www.timeanddate.com/worldclock/>

The workshop will take place in room D203 at the School of Mathematical Sciences, Hebei Normal University. Some talks will be online, and the workshop will be streamed via Zoom at the link

<https://us06web.zoom.us/j/86763384947?pwd=qXh0z0cHvaaiw2jqADI8iqNavdgm14.1>

The Zoom meeting will be open from 1:30 pm. The ID of the meeting is 86763384947, and the passcode is 612593.

# Abstracts

# Power Monoids in a New Framework for Factorization

Laura COSSU

*Abstract.* Let  $H$  be any multiplicative monoid, and consider the collection of all its finite subsets that contain the identity element. This collection forms a monoid under setwise multiplication, known as the reduced power monoid of  $H$  and denoted by  $\mathcal{P}_{\text{fin},1}(H)$ . Since  $\mathcal{P}_{\text{fin},1}(H)$  is non-cancellative whenever  $H$  is non-trivial, it serves as a central example in the study of a newly developed general theory of factorization. This theory, recently introduced by Cossu and Tringali, investigates decompositions into (almost) arbitrary factors within monoids that may admit non-trivial idempotents. Within this framework, we focus on minimal factorizations into irreducible elements in reduced power monoids. Among other results, we will discuss necessary and sufficient conditions on  $H$  under which  $\mathcal{P}_{\text{fin},1}(H)$  admits unique minimal factorizations.

## References

- [1] L. Cossu and S. Tringali, *On the arithmetic of power monoids*, J. Algebra **686** (Jan 2026), 793–813 (DOI: <https://doi.org/10.1016/j.jalgebra.2025.08.026>).

*About the speaker.* Laura Cossu is a tenure-track Assistant Professor in the Department of Mathematics and Computer Science at the University of Cagliari, Italy. She received her Ph.D. in Mathematics from the University of Padua (Italy) in 2017. She worked as a postdoctoral researcher in Padua and then at the University of Graz (Austria), where she was supported by various grants, including a Marie Skłodowska-Curie Individual Fellowship from the European Commission and a Principal Investigator project from the Austrian Science Fund. She has also been a visiting researcher at New Mexico State University (NM, USA), University of California, Irvine (CA, USA), Montclair State University (NJ, USA), and the Université Catholique de Louvain (Belgium). Her research interests include commutative and non-commutative ring and semigroup theory, factorization theory, and commutative algebra. She has published in various international journals, including J. London Math. Soc., Israel J. Math., and J. Algebra.

DIPARTIMENTO DI MATEMATICA E INFORMATICA, UNIVERSITÀ DEGLI STUDI DI CAGLIARI | VIA OSPEDALE 72, 09124 CAGLIARI, ITALY

*E-mail:* [laura.cossu3@unica.it](mailto:laura.cossu3@unica.it) · *Web page:* <https://sites.google.com/view/laura-cossu/home>

# Isomorphism problems for ideals of numerical semigroups

Pedro A. GARCÍA-SÁNCHEZ

*Abstract.* Let  $S$  be a numerical semigroup (a cofinite submonoid of the non-negative integers under addition). A non-empty set of integers  $I$  is said to be an ideal of  $S$  if  $I + S \subseteq I$  and  $I$  has a minimum. The sum of two ideals  $I$  and  $J$ , defined as  $I + J = \{i + j : i \in I, j \in J\}$ , is also an ideal of  $S$ . Thus, the set of ideals of  $S$ , denoted  $\mathfrak{I}(S)$ , is a commutative monoid under this operation, with neutral element  $S$ . If  $S$  and  $T$  are numerical semigroups and  $\mathfrak{I}(S)$  is isomorphic to  $\mathfrak{I}(T)$ , then  $S$  and  $T$  must be the same numerical semigroup [3].

If  $I$  and  $J$  are ideals of  $S$ , we write  $I \sim J$  if  $I = z + J$  for some  $z \in \mathbb{Z}$ . The ideal class monoid of  $S$  is defined as the set of ideals of  $S$  modulo this relation, where addition of two classes  $[I]$  and  $[J]$  is defined as  $[I] + [J] = [I + J]$ .

An ideal  $I$  is said to be normalized if  $\min(I) = 0$ . The set of normalized ideals of  $S$ , denoted by  $\mathfrak{I}_0(S)$ , is a monoid isomorphic to the ideal class monoid of  $S$  [1]. It is known that if  $S$  and  $T$  are numerical semigroups for which  $\mathfrak{I}_0(S)$  is isomorphic to  $\mathfrak{I}_0(T)$ , then  $S$  and  $T$  must be equal [2].

The set  $\mathfrak{I}_0(S)$  becomes a poset under inclusion. In [2], we also prove that if  $S$  and  $T$  are numerical semigroups such that the poset  $(\mathfrak{I}_0(S), \subseteq)$  is isomorphic to the poset  $(\mathfrak{I}_0(T), \subseteq)$ , then  $S = T$ .

On  $\mathfrak{I}_0(S)$  we can define a partial order  $\preceq$  as  $I \preceq J$  if there exists  $K \in \mathfrak{I}_0(S)$  such that  $I + K = J$ . We know that if  $S$  and  $T$  are numerical semigroups with multiplicity three such that the poset  $(\mathfrak{I}_0(S), \preceq)$  is isomorphic to the poset  $(\mathfrak{I}_0(T), \preceq)$ , then  $S$  and  $T$  are the same numerical semigroup [4]. However, if we remove the condition on the multiplicity, this poset isomorphism problem is still open.

In [5], we study the case when the poset  $(\mathfrak{I}_0(S), \preceq)$  is a lattice. We show that this is the case if and only if the multiplicity of  $S$  is at most four.

During the talk, I will give an overview of these recent results and present some open problems.

## References

- [1] L. Casabella, M. D'Anna, and P.A. García-Sánchez, *Apéry sets and the ideal class monoid of a numerical semigroup*, *Mediterr. J. Math.* **21** (2024), article No. 7.
- [2] P.A. García-Sánchez, *The isomorphism problem for ideal class monoids of numerical semigroups*, *Semigroup Forum* **108** (2024), 365–376.
- [3] P. A. García-Sánchez and S. Tringali, *Semigroups of ideals and isomorphism problems*, *Proc. Amer. Math. Soc.* **153** (2025), 2323–2339.
- [4] S. Bonzio and P.A. García-Sánchez, *The poset of normalized ideals of numerical semigroups with multiplicity three*, *Commun. Algebra* **53** (2025), 3610–3624.
- [5] S. Bonzio and P.A. García-Sánchez, *When the poset of the ideal class monoid of a numerical semigroup is a lattice*, preprint (<https://arxiv.org/abs/2412.07281>).

*About the speaker.* Pedro A. García-Sánchez defended his PhD thesis at the University of Granada (Spain) in 1996, where he has held a permanent position since 1999, became a full professor of algebra in 2017, and has served as the Director for Internationalization at the International School for Postgraduate Studies and coordinator of the Master in Mathematics. His primary research interests include numerical semigroups, commutative monoids, and non-unique factorization. He has served as the principal investigator for several research projects in these areas, and has authored or coauthored four books and approximately one hundred research papers. He is an associate editor of *Communications in Algebra* and has actively participated in various teaching innovation projects. He is currently vice-dean of Quality and Teaching Innovation at the Faculty of Sciences of the University of Granada.

DEPARTMENT OF ALGEBRA AND IMAG, UNIVERSITY OF GRANADA | 18071 GRANADA, SPAIN

*E-mail:* [pedro@ugr.es](mailto:pedro@ugr.es) · *Web page:* <https://www.ugr.es/local/pedro>

# Maximal Common Divisors in Power Monoids

Felix GOTTI

*Abstract.* A cancellative commutative monoid (resp., a commutative domain) is called *atomic* if every element (resp., non-zero element) factors into finitely many irreducibles, and it is called *irreducible-divisor-finite* (IDF) if every element (resp., non-zero element) has only finitely many irreducible divisors up to associates. In their landmark paper on factorization in integral domains (1990), Anderson, Anderson, and Zafrullah included the following two open questions:

- (1) Does the property of being an atomic domain ascend to polynomial rings?
- (2) Does the property of being an IDF domain ascend to polynomial rings?

A commutative monoid is said to have the *MCD property* if every non-empty finite subset admits a maximal common divisor (MCD). It is well known that every commutative monoid that satisfies the ascending chain condition on principal ideals (ACCP) necessarily has the MCD property. In 1993, Roitman gave a negative answer to Question 1 and proved that the property of being atomic does ascend to polynomial rings when restricted to the class of MCD commutative domains (a commutative domain is MCD if its multiplicative monoid is). The behavior of atomicity with respect to power monoids parallels this phenomenon, and we will discuss this in more detail during the first part of this talk.

On the other hand, we say that a commutative monoid has the *MCD-finite property* if every non-empty finite subset admits only finitely many MCDs up to associates. In 2009, Malcolmson and Okoh gave a negative answer to Question 2, proving that every countable commutative domain embeds into a countable commutative domain whose set of irreducibles is empty and whose polynomial ring does not have the IDF property. In 2018, Eftekhari and Khorsandi proved that the IDF property does ascend to polynomial rings when restricted to the class of MCD-finite domains. In the second part of this talk, we will examine the ascent of the IDF property to power monoids.

*About the speaker.* Felix Gotti is a lecturer and researcher at the Massachusetts Institute of Technology. He earned his PhD in Mathematics from UC Berkeley in 2019. His research primarily focuses on commutative algebra, semigroup theory, and combinatorics. He also serves as the research coordinator of MIT-PRIMES and as lead research mentor for CrowdMath, programs designed for highly motivated young students conducting mathematical research. He is an editor for Communications in Algebra and has published approximately 40 papers in a range of international journals, including J. Combin. Theory Ser. A, Proc. Amer. Math. Soc., and J. Algebra.

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY | 77 MASSACHUSETTS AVENUE, MA-02139-4307 CAMBRIDGE, UNITED STATES OF AMERICA

*E-mail:* fgotti@mit.edu · *Web page:* <https://math.mit.edu/~fgotti/>

# On power monoids and their automorphisms

Balint RAGO

*Abstract.* Let  $H$  be a multiplicatively written monoid. The family  $\mathcal{P}_{\text{fin},1}(H)$  of non-empty finite subsets of  $H$  containing the identity, endowed with the binary operation of setwise multiplication

$$(X, Y) \mapsto \{xy : x \in X, y \in Y\}$$

induced by  $H$ , is called the *reduced finitary power monoid* of  $H$ . Recently, Tringali and Yan [1] initiated the investigation of the automorphism group of these objects and showed that the reduced finitary power monoid of the monoid  $\mathbb{N}$  of non-negative integers under addition has precisely two automorphisms, the identity and the so-called *reversion map*. The existence of the latter is interesting in the sense that it is not the canonical extension of an automorphism of the base monoid  $\mathbb{N}$ .

In this talk, we give a complete description of the automorphism group of  $\mathcal{P}_{\text{fin},1}(H)$ , where  $H$  is either a finite abelian group or a submonoid of the additive group of rational numbers. More precisely, we show that there is a canonical isomorphism between the automorphism group of  $\mathcal{P}_{\text{fin},1}(H)$  and the automorphism group of  $H$ , except in certain special cases.

## References

- [1] S. Tringali and W. Yan, *On power monoids and their automorphisms*, J. Comb. Theory, Series A **209** (Jan 2025), article No. 105961, 16 pp.

*About the speaker.* Balint Rago is a 4th-year PhD student in the Discrete Mathematics Consortium of the Doctoral Academy at the University of Graz (Austria). His research focuses on commutative ring theory and factorization theory, with a recent interest in the study of power semigroups and power monoids. Some of his results have been published in journals such as Proc. Amer. Math. Soc., Pacific J. Math., and Acta Arith.

DEPARTMENT OF MATHEMATICS AND SCIENTIFIC COMPUTING, NAWI GRAZ | HEINRICHSTRASSE 36, 8010 GRAZ, AUSTRIA

*E-mail:* balint.rago@uni-graz.at

# Power monoids and their system of length sets

Andreas REINHART

*Abstract.* Let  $\mathcal{P}_{\text{fin},0}(\mathbb{N})$  be the family of all non-empty finite subsets of  $\mathbb{N}$  that contain 0, where  $\mathbb{N}$  is the set of non-negative integers. Equipped with the binary operation of set addition

$$(A, B) \mapsto A + B := \{a + b : a \in A, b \in B\},$$

this family forms a commutative monoid, called the **reduced finitary power monoid** of  $\mathbb{N}$ .

The identity element is the singleton  $\{0\}$ . A non-identity element  $A \in \mathcal{P}_{\text{fin},0}(\mathbb{N})$  is called an **atom** if it cannot be expressed as the sum of two elements of  $\mathcal{P}_{\text{fin},0}(\mathbb{N})$ , both distinct from  $\{0\}$ . For each  $B \in \mathcal{P}_{\text{fin},0}(\mathbb{N})$ , we denote by  $\mathsf{L}(B)$  the **length set** of  $B$ , that is, the set of all integers  $n \geq 0$  such that  $B$  can be written as the sum of  $n$  atoms of  $\mathcal{P}_{\text{fin},0}(\mathbb{N})$ . The collection

$$\mathcal{L}(\mathcal{P}_{\text{fin},0}(\mathbb{N})) := \{\mathsf{L}(B) : B \in \mathcal{P}_{\text{fin},0}(\mathbb{N})\}$$

is called the **system of length sets** of  $\mathcal{P}_{\text{fin},0}(\mathbb{N})$ .

It is not difficult to verify that every length set of  $\mathcal{P}_{\text{fin},0}(\mathbb{N})$ , except for  $\{0\}$  and  $\{1\}$ , is a non-empty finite subset  $L$  of  $\mathbb{N}$  whose minimum is larger than or equal to 2. Fan and Tringali (2018) conjectured that the converse also holds, namely, every such set  $L$  is indeed a length set of  $\mathcal{P}_{\text{fin},0}(\mathbb{N})$ .

In this talk, we present several results supporting this conjecture. In particular, we show that  $\mathcal{P}_{\text{fin},0}(\mathbb{N}_0)$  is **fully elastic**, that is, for every rational number  $r \geq 1$ , there exists a set  $C \in \mathcal{P}_{\text{fin},0}(\mathbb{N}_0)$ , distinct from  $\{0\}$ , such that  $\max \mathsf{L}(C) = r \min \mathsf{L}(C)$ .

## References

- [1] Y. Fan and S. Tringali, *Power monoids: A bridge between factorization theory and arithmetic combinatorics*, J. Algebra **512** (Oct 2018), 252–294.

*About the speaker.* Andreas Reinhart is currently a postdoctoral researcher at the University of Graz. He earned his Ph.D. under the supervision of Professor Franz Halter-Koch in 2010 and completed his habilitation at the University of Graz in 2020. His research focuses on various topics in commutative ring theory, factorization theory, multiplicative ideal theory, and algebraic number theory. He has published approximately 25 research papers in a range of academic journals.

DEPARTMENT OF MATHEMATICS AND SCIENTIFIC COMPUTING, NAWI GRAZ | HEINRICHSTRASSE 36, 8010 GRAZ, AUSTRIA

*E-mail:* andreas.reinhart@uni-graz.at · *Web page:* <http://imsc.uni-graz.at/reinhart/>

# Recent trends in the theory of power semigroups

Salvatore TRINGALI

*Abstract.* Let  $S$  be a multiplicatively written semigroup. Endowed with the operation of setwise multiplication induced from  $S$  and defined by

$$(X, Y) \mapsto \{xy : x \in X, y \in Y\},$$

the family of non-empty subsets of  $S$  forms a semigroup in its own right, hereafter denoted by  $\mathcal{P}(S)$ .

The term *power semigroup* is generically used for various subsemigroups of  $\mathcal{P}(S)$  that, in a certain vague sense, lie between  $S$  and  $\mathcal{P}(S)$  itself. I will review recent progress on power semigroups and highlight some questions and open problems that are currently guiding the evolution of the theory.

## References

- [1] A.A. Antoniou and S. Tringali, *On the Arithmetic of Power Monoids and Sumsets in Cyclic Groups*, Pacific J. Math. **312** (2021), No. 2, 279–308.
- [2] P.-Y. Bienvenu and A. Geroldinger, *On algebraic properties of power monoids of numerical monoids*, Israel J. Math. **265** (2025), 867–900.
- [3] L. Cossu and S. Tringali, *On the arithmetic of power monoids*, J. Algebra **686** (Jan 2026), 793–813 (DOI: <https://doi.org/10.1016/j.jalgebra.2025.08.026>).
- [4] Y. Fan and S. Tringali, *Power monoids: A bridge between factorization theory and arithmetic combinatorics*, J. Algebra **512** (Oct 2018), 252–294.
- [5] P.A. García-Sánchez and S. Tringali, *On semigroups of ideals and related isomorphism problems*, Proc. Amer. Math. Soc. **153** (2025), No. 6, 2323–2339.
- [6] B. Rago, *A counterexample to an isomorphism problem for power monoids*, Proc. Amer. Math. Soc. (202?), to appear.
- [7] S. Tringali and W. Yan, *On power monoids and their automorphisms*, J. Comb. Theory, Series A **209** (Jan 2025), #105961, 16 pp.
- [8] S. Tringali and W. Yan, *A conjecture of Bienvenu and Geroldinger on power monoids*, Proc. Amer. Math. Soc. **153** (2025), No. 3, 913–919.

*About the speaker.* Salvatore Tringali holds a PhD in mathematics from the University of Lyon (Jean Monnet campus) in France and a PhD in electronic engineering from the University Mediterranea in Italy. Prior to joining the School of Mathematical Sciences at Hebei Normal University in Feb 2019, he was a visiting professor at Nankai University (Sep–Dec 2018), a Lise Meitner fellow (Mar 2016–Feb 2018) and a lecturer (Mar–Aug 2018) at the University of Graz in Austria, a research associate at Texas A&M University at Doha in Qatar (Nov 2014–Oct 2015), and a postdoc at the École polytechnique in France (Nov 2015–Feb 2016 and Jan–July 2014).

He has authored or co-authored approximately 40 research papers, some published in international journals such as Math. Proc. Cambridge Phil. Soc., Proc. Amer. Math. Soc., J. Comb. Theory Ser. A, J. Algebra, Pacif. J. Math., and Israel J. Math. His primary research interests lie at the intersection of algebra, combinatorics, and number theory.

SCHOOL OF MATHEMATICAL SCIENCES, HEBEI NORMAL UNIVERSITY | SHIJIAZHUANG, HEBEI PROVINCE, 050024 CHINA

*E-mail:* [salvo.tringali@gmail.com](mailto:salvo.tringali@gmail.com)